Volaris Capital Management - Fallacy of Risk-Neutral Approach to Options

Vivek Kapoor,
CEO/CIO
Margaret Sundberg,
Quantitative Trader & Portfolio Manager
Volaris Capital Management, LLC

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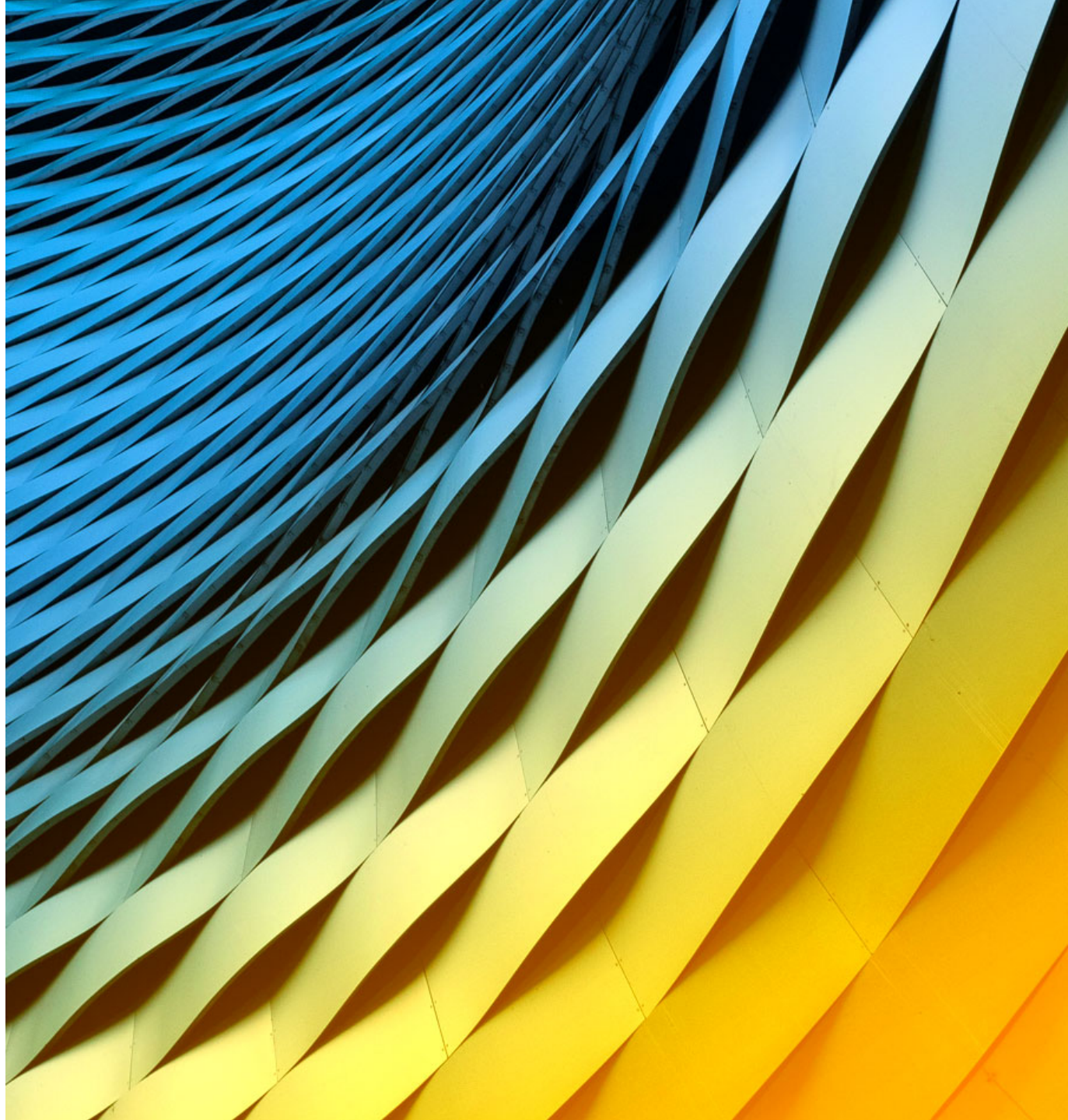

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Irreducible Risks:

Fallacy of Risk-Neutral Approach to Options

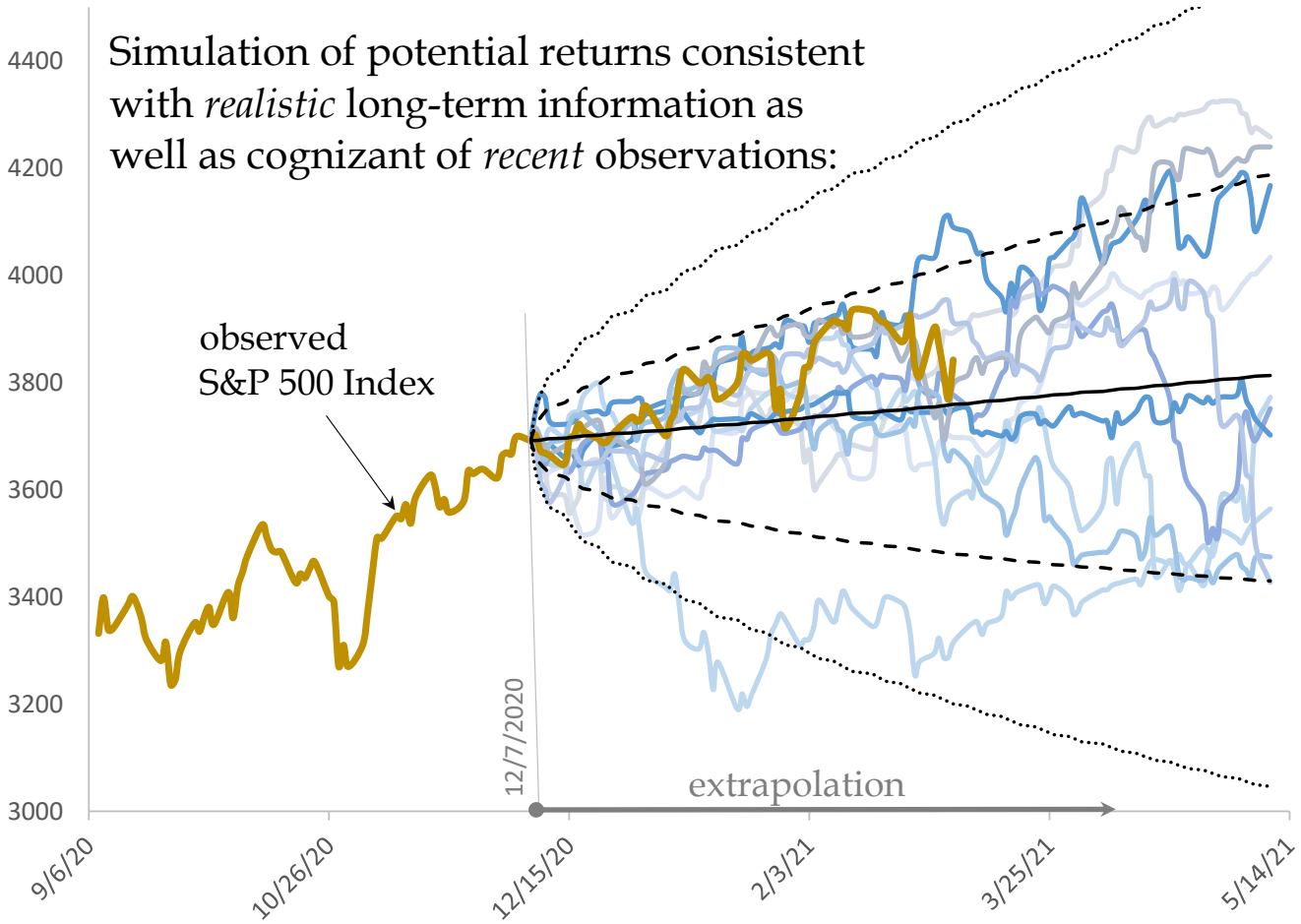
Vivek Kapoor

Chief Investment Officer

Margaret Francis Sundberg

Quantitative Trader & Portfolio Manager

VOLARIS CAPITAL MANAGEMENT



Take calculated risks. That is quite different from being rash.
General George Patton

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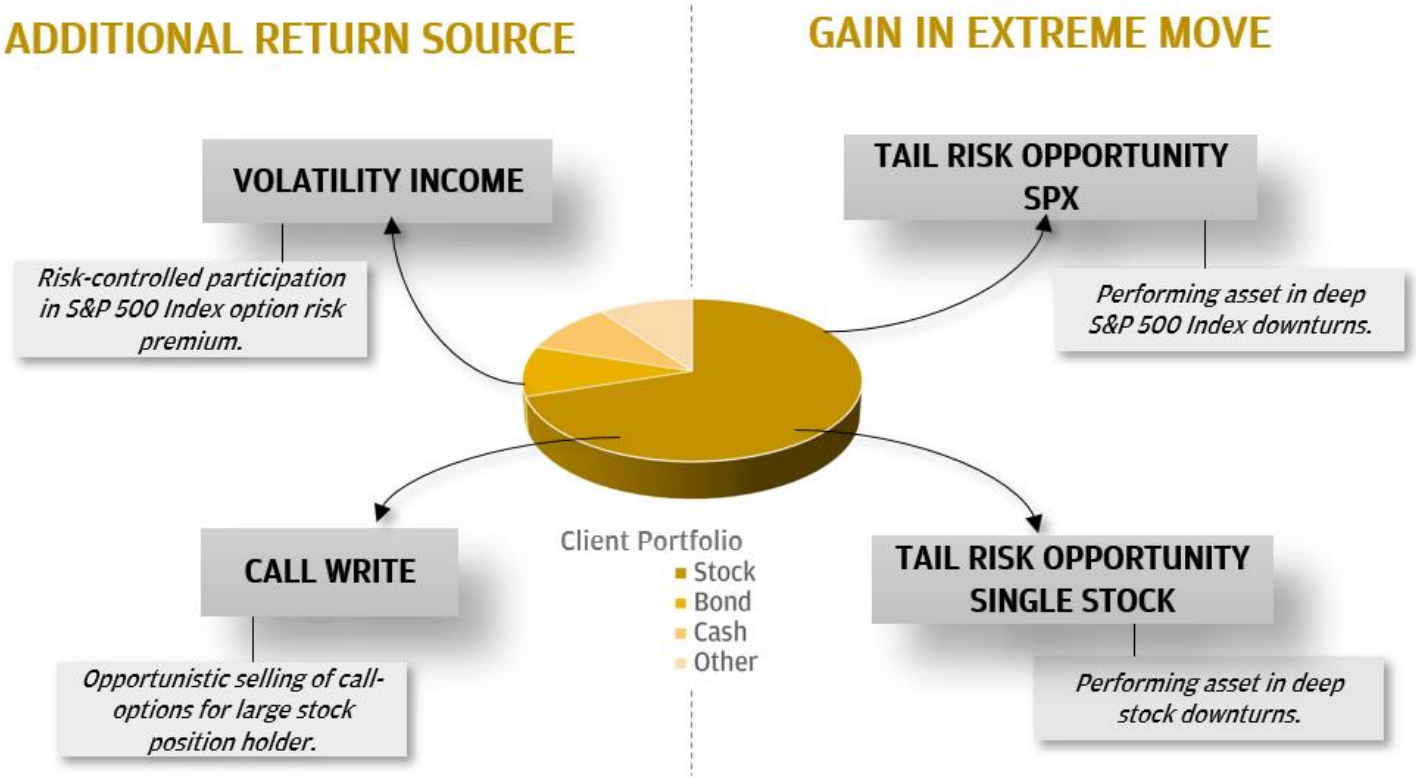
Volaris Mission

- *We provide investors access to yield and tail-risk opportunities in the options market.*
- *Our insights, based on experience and active research, are applied to pursue client's goals.*

VOLARIS
CAPITAL MANAGEMENT

Volaris Solutions

- Clients may use existing portfolio holding (stocks, bonds, etc.) as collateral to employ Volaris strategies.
- Minimal opportunity cost implementation employing listed options in Separately Managed Account.
- Defined risk strategies with transparency and daily liquidity.



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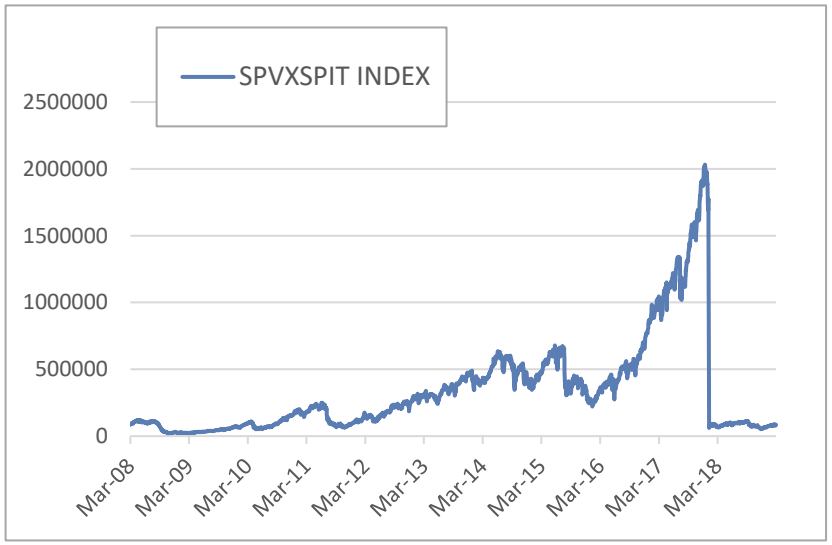
Failure of Indexation in Volatility: *Exhibit A*

SHORT VOLATILITY

Going broke selling Volatility

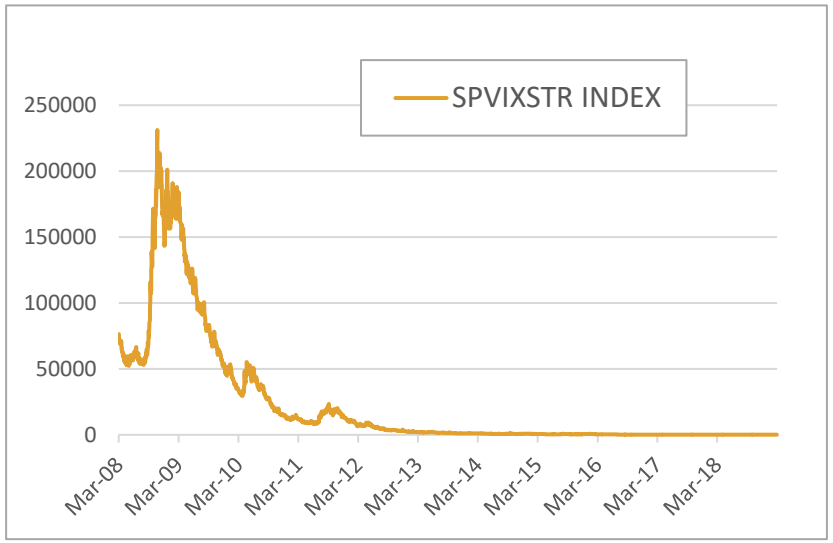
SPVXSPIT Index (S&P 500 VIX Short-Term Futures Inverse Index Total Return): Represents short exposure to one-month maturity VIX futures contracts.



LONG VOLATILITY

Going broke buying Volatility

SPVIXSTR Index (S&P 500 VIX Short-Term Futures Index Total Return): Represents long exposure to one-month maturity VIX futures contracts.



VIX futures or Variance Swaps do not lend to a prudential portfolio allocation.

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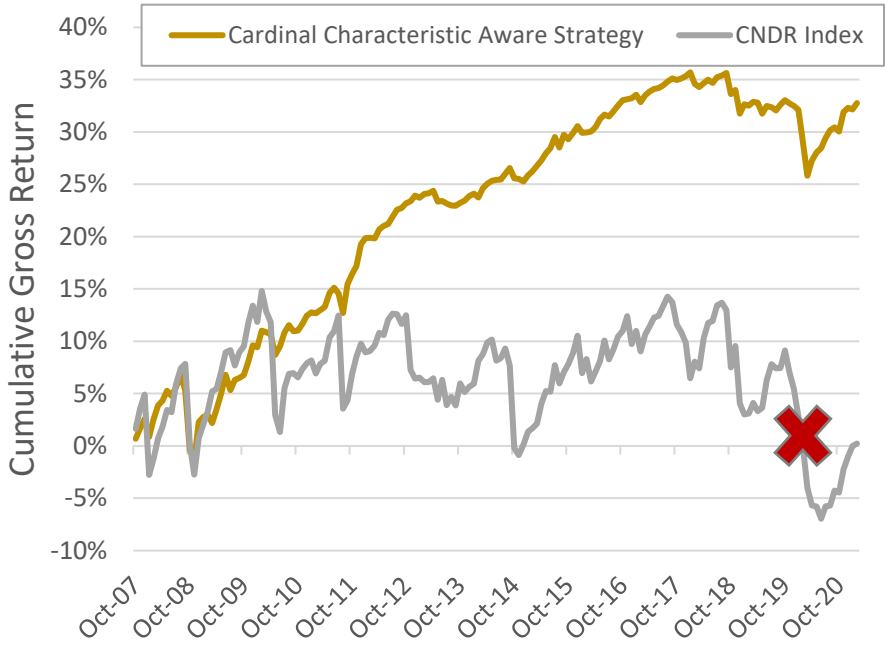
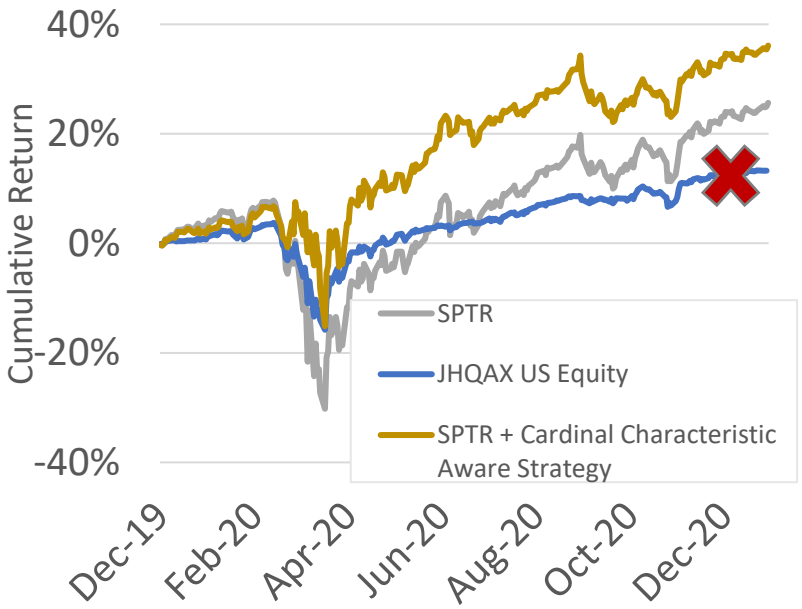
Failure of Indexation in Volatility: *Exhibit B*





Five Cardinal Characteristics of SPX Index Options
1. *Disparity Between Out of the Money Puts and Calls*
2. *Asymmetry Between Buyer and Seller*
3. *Exploding Asymmetry at Expiry*
4. *Exploding Asymmetry with Out-of-the-Moneyness*
5. *Elements of Timing (recognizing Volatility environment)*

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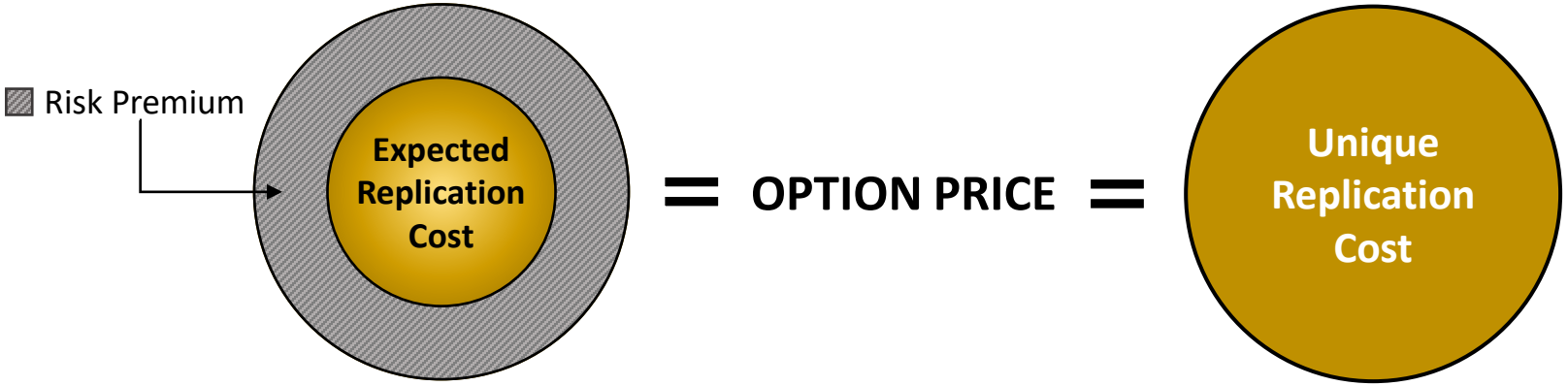
Risk-Aware vs. Risk-Neutral



Risk Aware Approach

- Requires assessing attempted replication costs.
- Expected attempted replication costs of a sold option have a highly adverse asymmetric distribution around them.
- Deviations of option price from expected attempted replication costs are *interpreted* as risk-premiums.

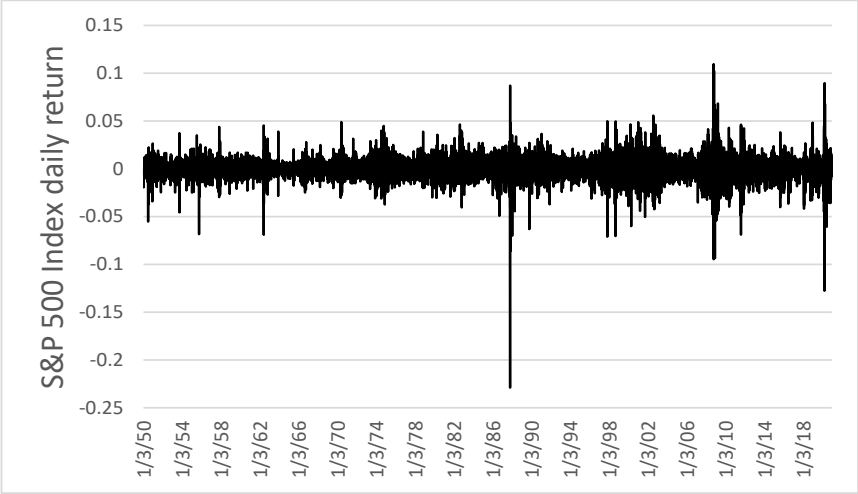
Risk Neutral Approach

- Makes no explicit reference to irreducible attempted replication uncertainty.
- Assumes option prices are an expected option payoff under a "risk-neutral-measure".
- Risk-neutral-measures are to be *fitted* to observed option prices.

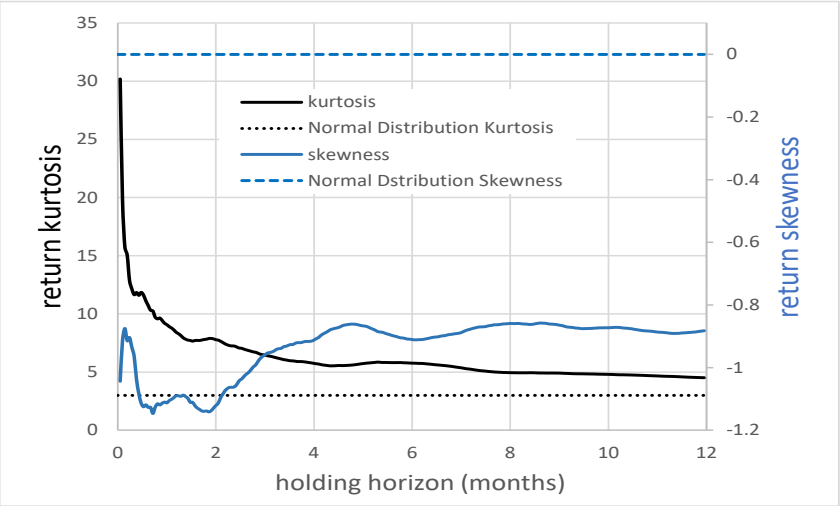
"Lead me from unreal to real". Brihadaranayaka Upanishad 700 BC

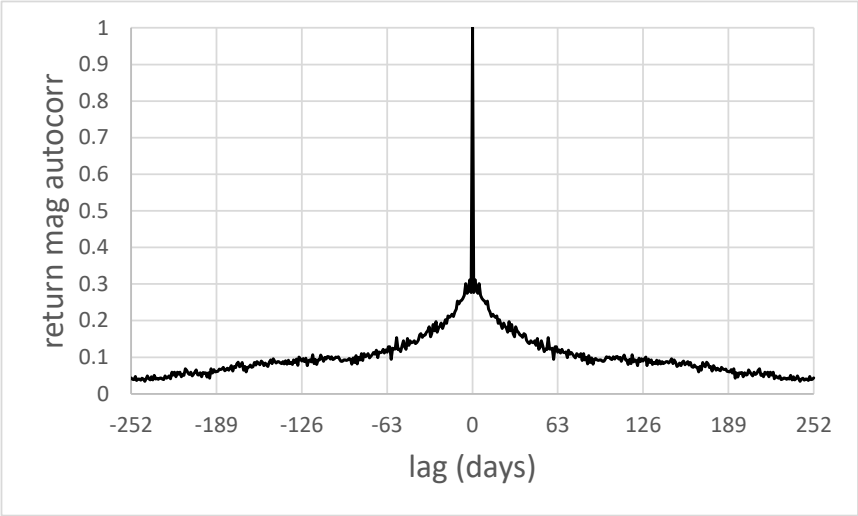
Real-Returns

RYTHMS & RHYMES

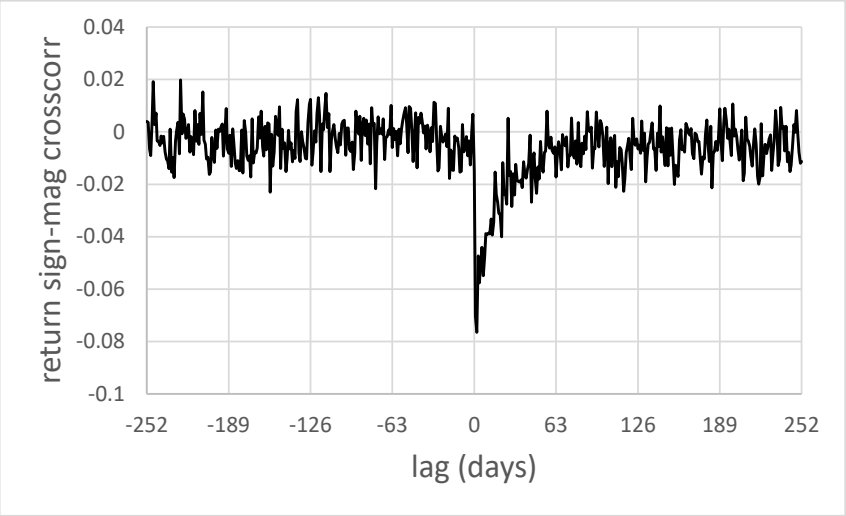


NON-NORMALITY





PERSISTENCE



ASYMMETRY

"Molecules colliding randomly under thermal agitations have nothing in common with the human enterprise of risk-taking, trading, and price discovery".

Anonymous 2000 AD

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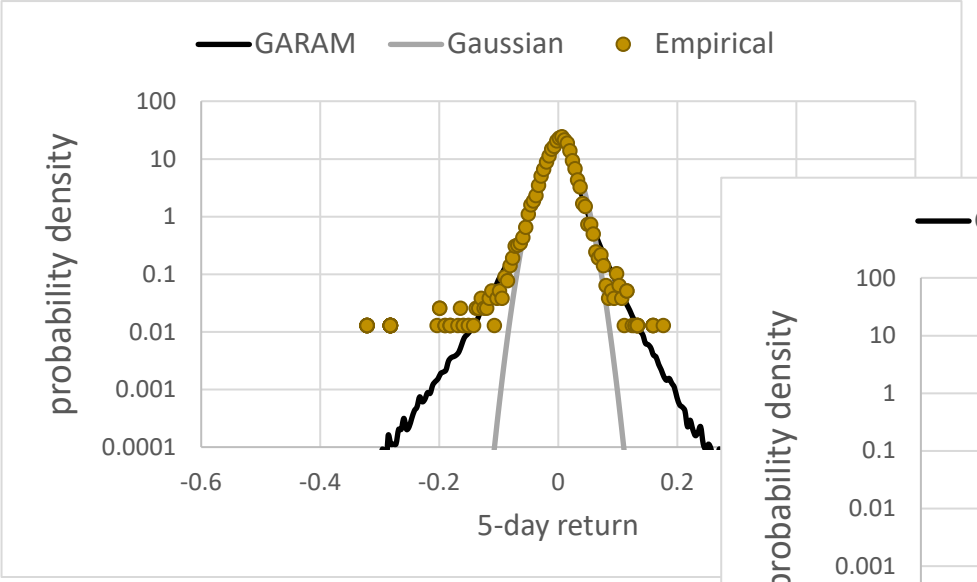
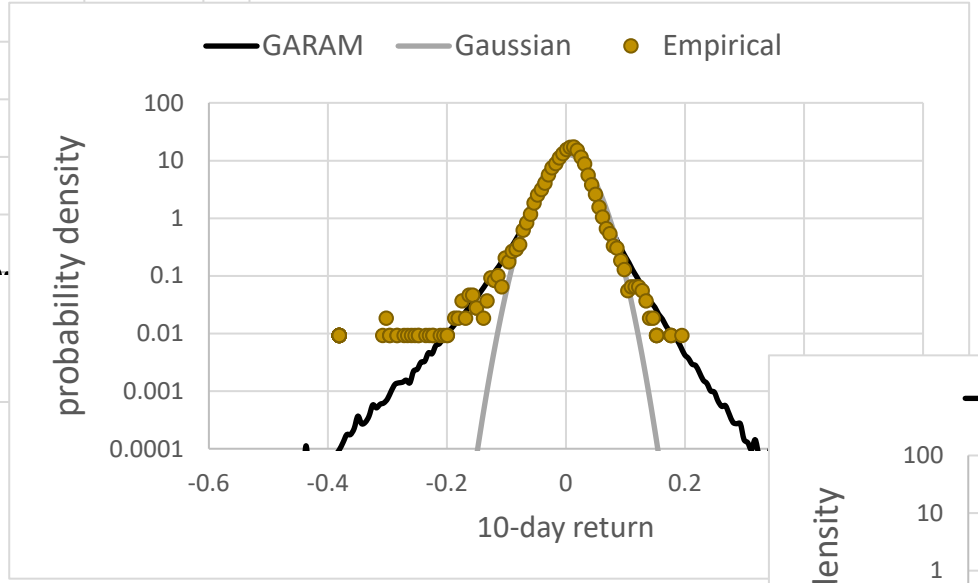
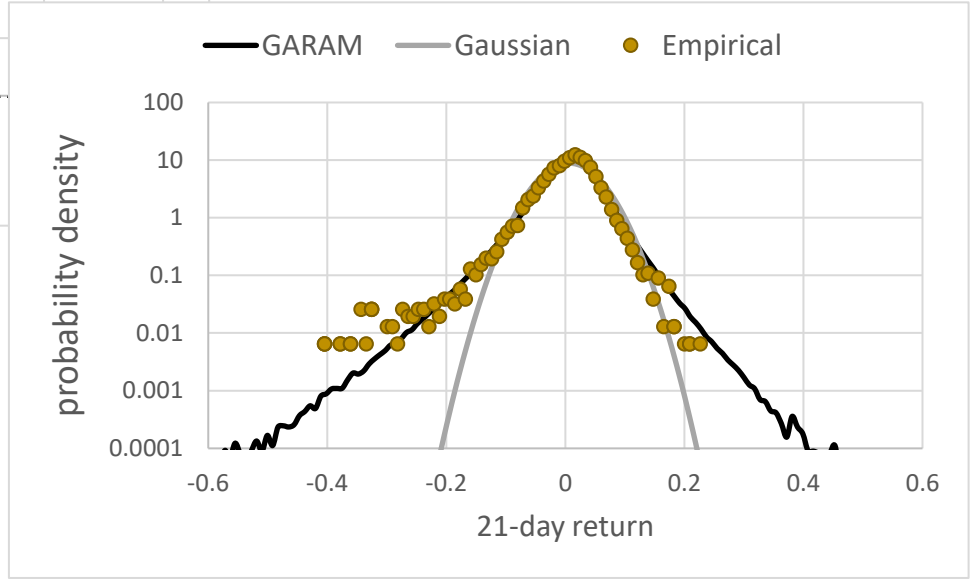
Real Returns

Term-Structure of the Non-Gaussianity of S&P 500 Index Returns







Holding horizon (i.e., term)

→

Buildup of asymmetry and persistence of kurtosis

Wang, Jiaxin and Petrelli, Andrea and Balachandran, Ram and Siu, Olivia and Zhang, Jun and Chatterjee, Rupak and Kapoor, Vivek, **General Auto-Regressive Asset Model (July 1, 2009)**. Available at SSRN: https://ssrn.com/abstract=1428555 or http://dx.doi.org/10.2139/ssrn.1428555

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Volaris Risk-Aware Approach

Our tool to describe the asset is a ***Real-World Stochastic Model***:

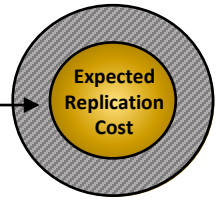
- We utilize observed persistence & lead-lag relationships between return magnitude and sign and other *conditioning variables* using a *vector auto-regressive* framework to realistically capture the first four moments of return term structure.

Our tool for analyzing options is ***Multi-Variate Variational Calculus***:

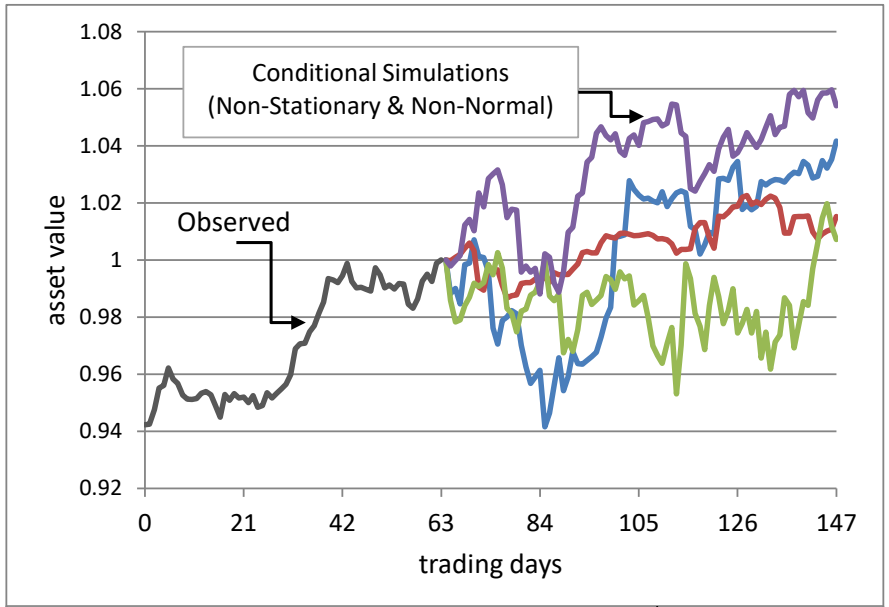
- The pair of *optimal functions* being sought are the hedge-ratio and expected attempted replication cost as a function of the underlying. The functional being minimized is the conditional wealth change variance.
- An explicit articulation of the optimal hedging strategy at each time-step enables an *ex-ante* assessment of residual risks.

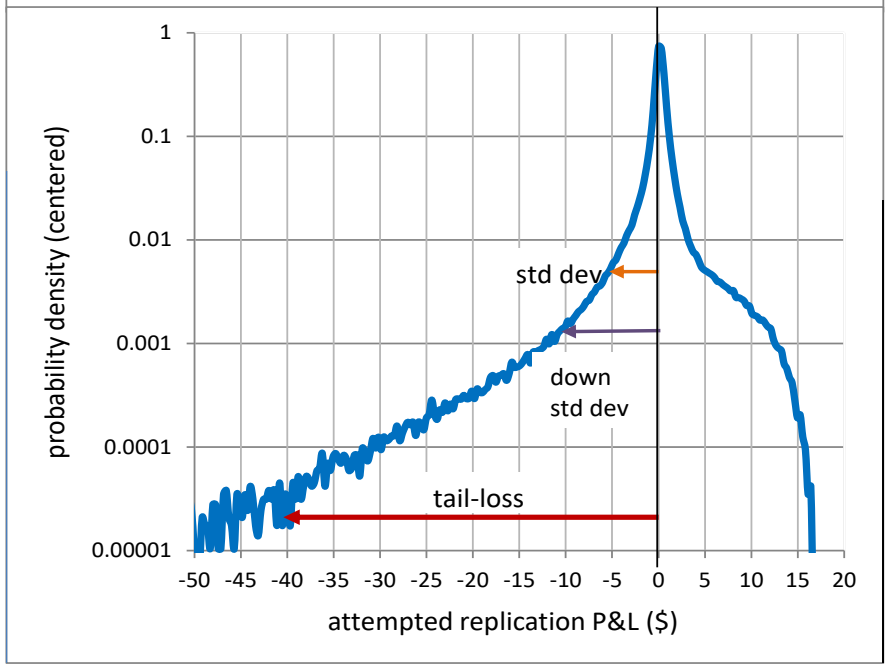
We infer Option Risk Premiums by comparing price with conditional Expected Costs of Attempted Replication.

$$\underset{\text{option-price}}{C} = \underbrace{\overline{G} - \overline{\Delta W^h}}_{\text{expected replication cost}} + \underset{\text{risk-premium}}{\overline{\Delta W}}$$



G : *option payoff* $\quad \Delta W^h$: *hedge P & L*





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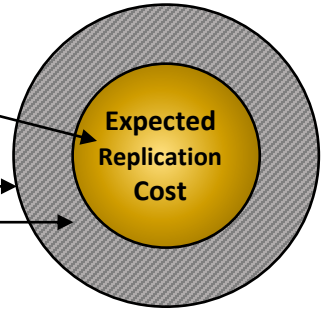
Inference of Option Risk Premium

Expected P&L of Attempted Replication

$$\underset{risk-premium}{\overline{\Delta W}} = \underset{option\text{-}price}{C} - \Big[\underbrace{\overline{G} - \overline{\Delta W^h}}_{expected\ replication\ cost}\Big]$$



$G: option\ payoff \qquad \Delta W^h:\ hedge\ P\&L$

Attempted Replication P&L Fluctuations

Irreducible uncertainty arount attempted replication cost: $\Delta W'$

Risk-Capital (Q) at confidence level p_s: Probability $\{\Delta W' > -Q\} = p_S$

Attempted P&L replication standard deviation: $\sigma_{\Delta W}$

Attempted P&L replication downside standard deviation: $\sigma^-_{\Delta W}$

Option Price Implied Risk Premium Metrics

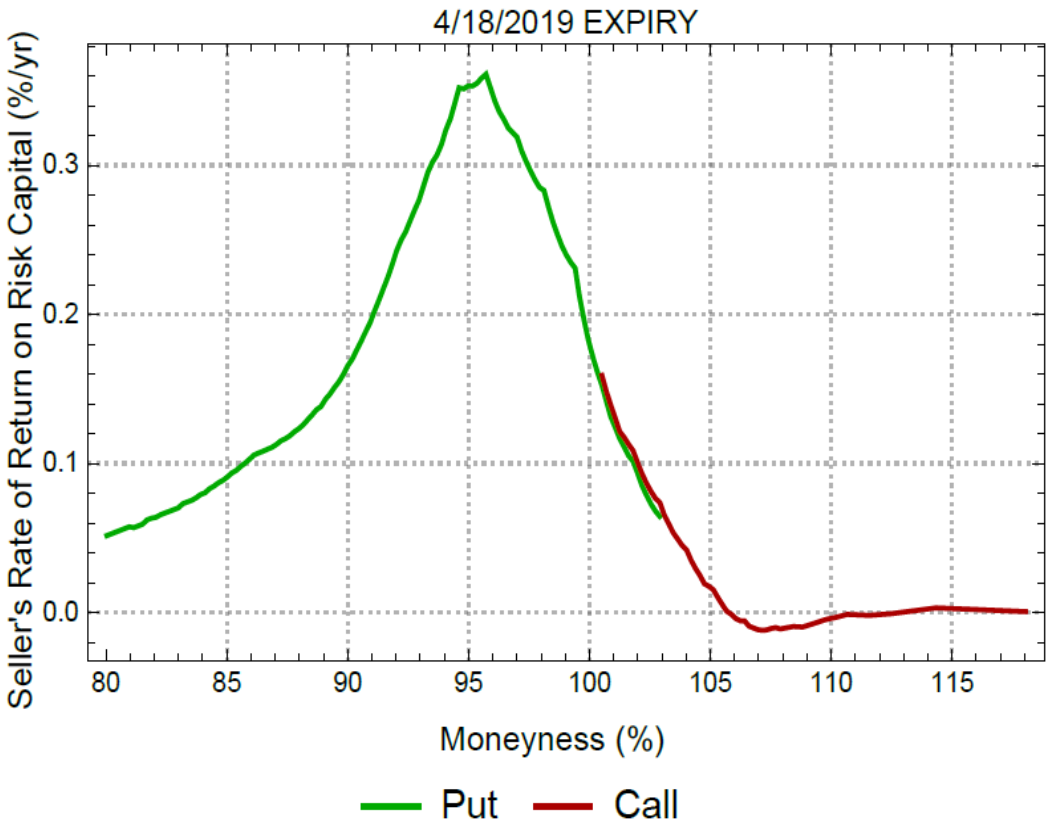
Sortino Ratio: $\Lambda^- = \dfrac{\overline{\Delta W}}{\sigma^-_{\Delta W} \times \sqrt{T}}$

Information Ratio: $\Lambda = \dfrac{\overline{\Delta W}}{\sigma_{\Delta W} \times \sqrt{T}}$

Rate of Expected Return on Capital: $\Theta = \ln\left[\dfrac{\overline{\Delta W}}{Q} + 1\right] / T$

By quantifying the distribution of possible replication costs the price of an option implies a metric of expected return per unit risk.

Sample Inference of S&P 500 Index Option Risk Premium



A sample snapshot is shown from February 11, 2019 of the estimated risk-return profile of an option seller.

Comparison of an option *bid-price* with expected replication costs provides an estimate of the option-seller's *expected P&L*. The *expected P&L* when compared to the *replication-cost distribution* reveals the option's risk-return profile from a seller's perspective.

Sellers Rate of Expected Return on Risk Capital

$$\theta(T) = \frac{\ln\left[\frac{\overline{\Delta W}}{Q} + 1\right]}{T} \text{(1/year)}$$

$$\overline{\Delta W} = \{\text{Bid Price}\} - \{\text{Expected Attempted Replication Cost}\}$$

$$Q = Risk\ \ Capital$$

T: duration of derivative (years)

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Risk-Neutral Approach?

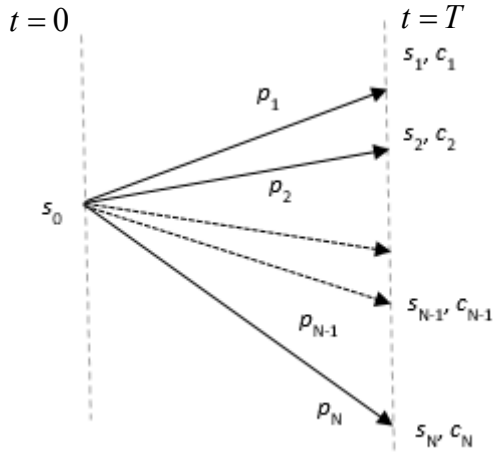
What does the risk-neutral approach mean in the presence of irreducible risks and associated risk-premiums?

OPTION PRICE = **Unique Replication Cost** **=** **EXPECTATION OF OPTION PAYOFF UNDER RISK NEUTRAL MEASURE**

- What is the meaning of the option price fitted probability measure in the presence of risk premiums?

- Can fitted risk-neutral measures accommodate risk-premiums?

Risk-Aware Approach vs. Risk-Neutral Approach?



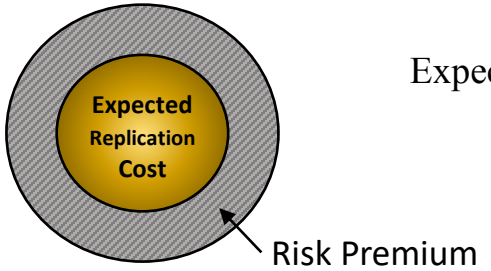
Discrete State Static Hedging

Risk-Aware Approach:

ϕ: Hedge ratio

G: Option payoff

ΔW: P&L



Optimal Hedge Ratio: $\phi_{opt} = \dfrac{E[Gs_T] - \bar{G}\,\bar{s}_T}{\sigma^2_{s_T}}$

Irreducible P&L Variance: $\sigma^2_{\Delta W}\big|_{\min} = \sigma^2_G - \dfrac{\left(E[Gs_T] - \bar{G}\,\bar{s}_T\right)^2}{\sigma^2_{s_T}}$

Expected Replication Cost: $\chi = \bar{G} - \dfrac{(\bar{s}_T - s_0)}{\sigma^2_{s_T}}\left(\overline{Gs_T} - \bar{G}\bar{s}_T\right)$

Model Option Price: $C = \chi + \lambda\left[\sigma^2_{\Delta W}\big|_{\min}\right]^{1/2}$

Expected Replication Cost:

$$\exists\, \hat{p}_i \ni \chi = \hat{E}[G] \text{ and } \hat{E}\left[s_T | s_0\right] = s_0, \text{ where } \hat{E}[.] \text{ is an expectation under } \hat{p}_i \text{ with } \sum_{i=1}^{N} \hat{p}_i = 1.$$

$$\hat{p}_i = p_i \times \left\{1 - \frac{(\bar{s}_T - s_0)(s_i - \bar{s}_T)}{\sigma^2_{s_T}}\right\}$$

Model Option Price:

$$\exists\, \tilde{p}_i \ni C = \tilde{E}[G] \text{ and } \tilde{E}\left[s_T | s_0\right] = s_0, \text{ where } \tilde{E}[.] \text{ is an expectation under } \tilde{p}_i, \text{ and } \sum_{i=1}^{N} \tilde{p}_i = 1.$$

Unique Replication Cost

$$\tilde{p}_i = \hat{p}_i + \dot{p}_i; \quad \dot{p}_i = \frac{\lambda}{2\sigma_{\Delta W}\big|_{\min}}\left\{2p_i c_i - 2p_i\bar{G} - \frac{2\left(E[Gs_T] - \bar{G}\,\bar{s}_T\right)\left(p_i s_i - p_i \bar{s}_T\right)}{\sigma^2_{s_T}}\right\}$$

Risk-Neutral Measure: Existence & Uniqueness

Two-State Single Period Hedging: The Trivial Case

Asset Characteristics:

Initial ($t = 0$) asset value $s_0 = 100$ Final ($t = T$) asset value $s_1 = 110$, $s_2 = 80$

$p_1 = 0.95$, $p_2 = 0.05$; $E[s_T] = 108.5$, $\sigma_{s_T} = 6.53835$

$\hat{p}_1 = 0.\overline{6}$, $\hat{p}_2 = 0.\overline{3}$; $\hat{E}[s_T] = 100$, $\hat{\sigma}_{s_T} = 14.1421$

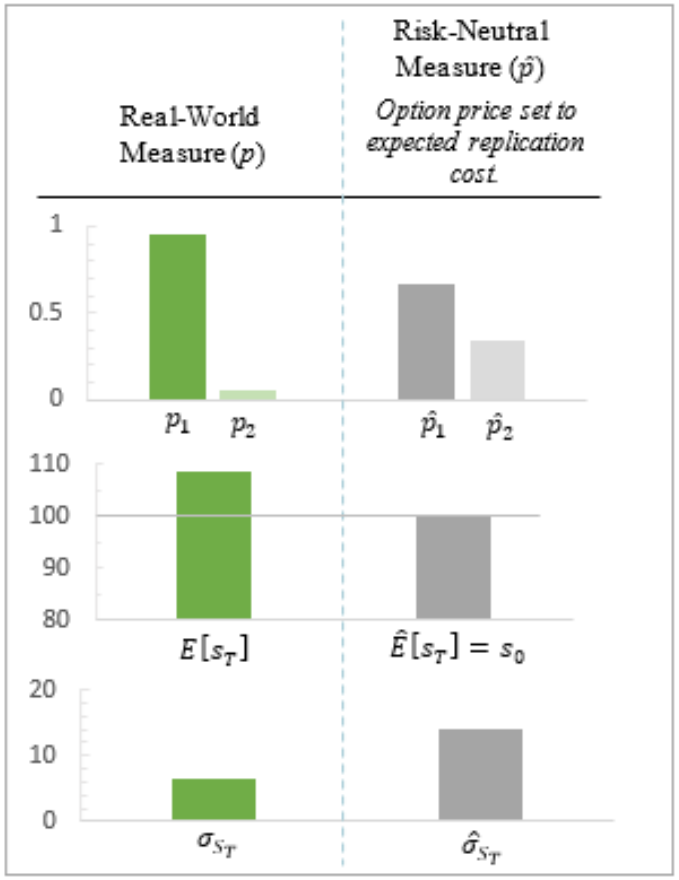
Option Contract: 85 Strike Put; $\{c_1 = 0,\ c_2 = 5\}$

Hedging Strategy

P&L variance optional hedge ratio $\phi_{opt} = -0.1\overline{6}$

Expected attempted replication cost $\chi = 1.\overline{6}$

Irreducible P&L standard deviation $\sigma_{\Delta W}\big|_{\min} = 0$

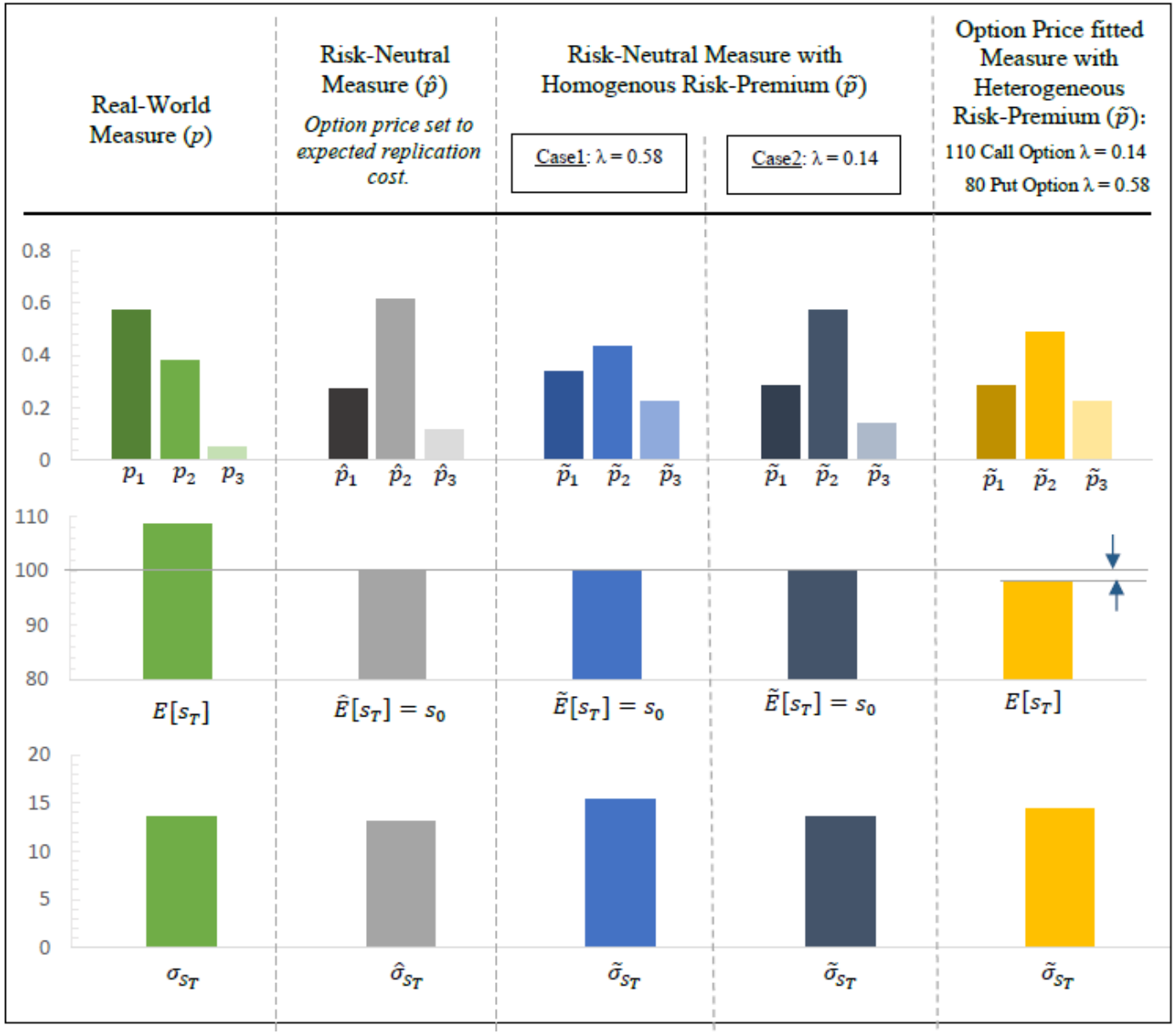


Two-State Perfect Static Hedge Numerical Example.

The expected attempted replication cost can be represented as an expectation under a risk-neutral measure without any change in expected value for an investment in the underlying.

The hedge ratio is independent of real-world probability and the residual risk is eliminated.

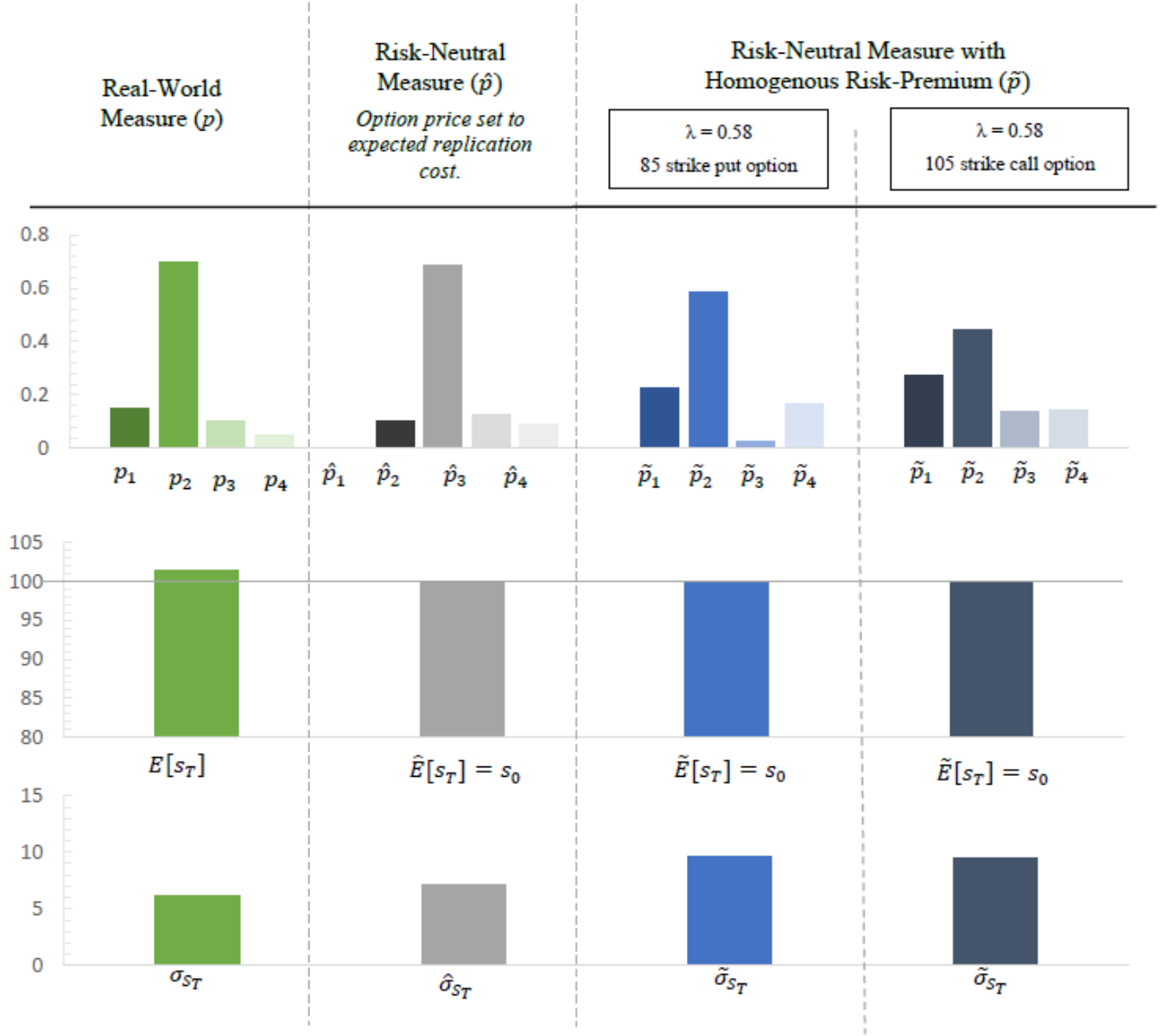
Three-State Static Hedging Examples



Three State Static Hedging Example.

The expected attempted replication cost can be represented as an expectation under a risk-neutral measure without any change in expected value for an investment in the underlying. Assuming the option prices to be the expected hedging costs plus a homogeneous fraction of the irreducible P&L standard deviation, they can also be represented as an expectation under a risk-neutral measure without any change in expected value for an investment in the underlying. The optimal hedge ratio and risk-neutral probabilities are functions of real-world probability and the irreducible risk is not identically zero – unlike the special two-state asset afforded. Under heterogeneous risk premiums one fails to find a risk neutral measure that fits the option price and render an investment in the asset to be static under expectations.

Four-State Static Hedging Examples



Four State Static Hedging Example.
The expected attempted replication cost can be represented as an expectation under a risk-neutral measure without any change in expected value for an investment in the underlying. Assuming the option prices to be the expected hedging costs plus a homogeneous fraction of the irreducible P&L standard deviation, they can also be represented as an expectation under a risk-neutral measure without any change in expected value for an investment in the underlying. The optimal hedge ratio and risk-neutral probabilities are functions of real-world probability and the irreducible risk is not identically zero – unlike the special two-state asset afforded. Even under a homogeneous risk premium the implied risk neutral measure is not unique.

Risk Aware vs. Risk Neutral

Risk Aware Treatment of Options

- One finds that most option contracts involve hedge slippage that is at least the same order of magnitude as the average attempted replication cost.
- A risk aware paradigm enables interpreting option prices in terms of expected attempted replication costs and risk premiums.

Risk Neutral Treatment of Options

- Fitted risk-neutral probabilities, as such, do not shed light on risk premiums.
- On account of irreducible risks, a *unique risk-neutral probability measure* fails to exist.

- Historical Mistake
- Facilitates Heist of Risk-Capital
  - *OTC derivatives warehoused without commensurate risk capital*
  - *Periodic uncontrolled blow-ups*
- Dysfunction due to a confluence of unrealistic Financial Economics, "Physics," and "Mathematics"

"Doubt is not a pleasant condition, but certainty is absurd."

Voltaire

Risk-Return Awareness

Five Cardinal Characteristics of SPX Index Options
1. *Disparity Between Out of the Money Puts and Calls*
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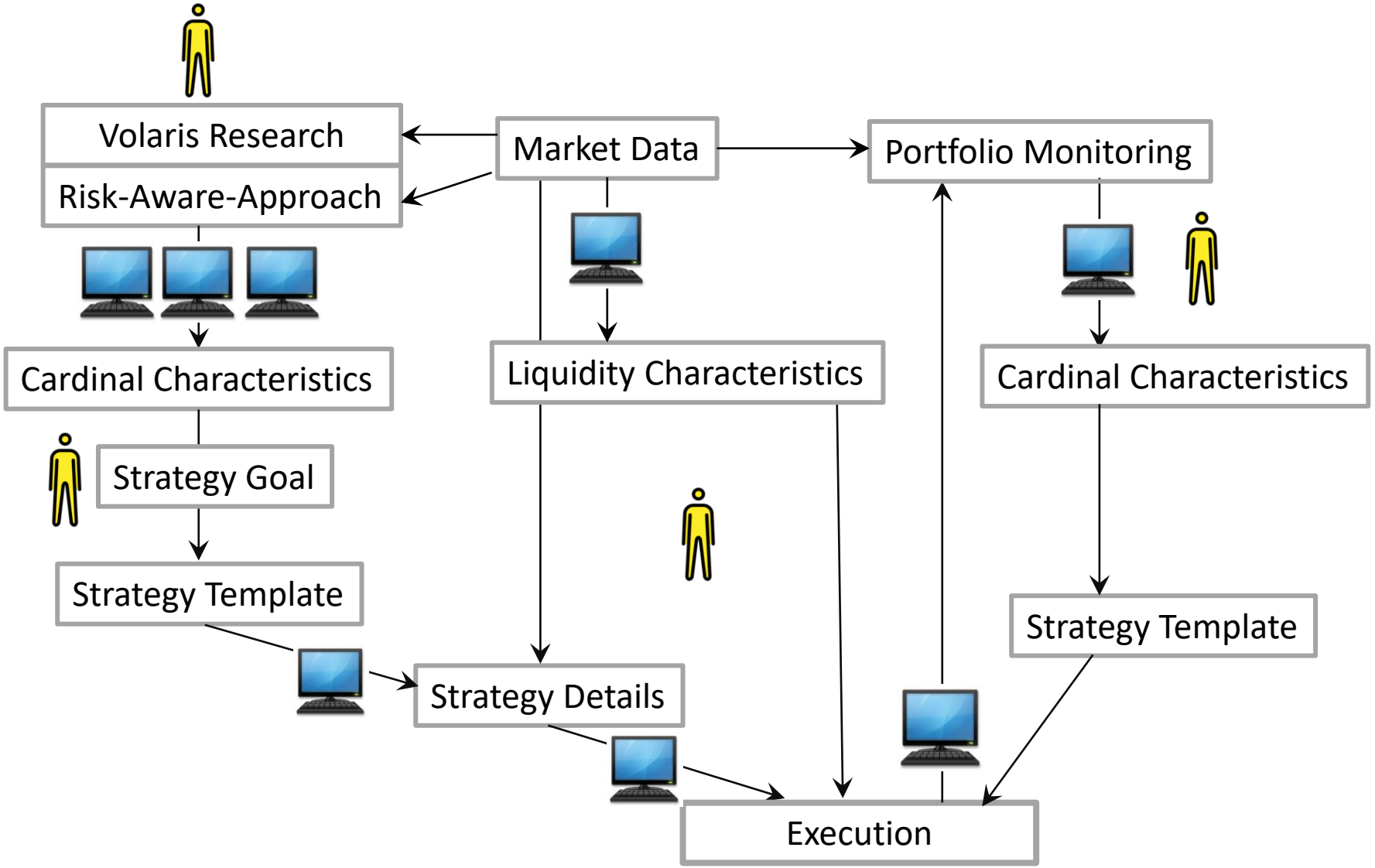
Volaris Strategy Design-Execution

Emphasis

- *Focus on Human Learning*
- *Data Driven Real World Analysis*
- *Clearly Articulated Models*
- *Awareness of Asymmetry of Options*
- *Efficient Computation*
- *Real-Time Option Risk-Return Articulation*
- *Articulation of Risk Preference/Aversion*
- *Simple Instruments*
- *Well Defined Risk Profiles*
- *Intelligent Automation*



Contacts & References

Vivek Kapoor
CEO/CIO
vivek.kapoor@volariscapital.com
1-973-379-0600

Margaret Sundberg
Portfolio Manager
margaret.sundberg@volariscapital.com
1-973-379-0604

References:

- https://ssrn.com/abstract=3761304
- Research (volariscapital.com)

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